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To: Dr. D. Mills
Director of Research

OPTIMAL PREVENTIVE MAINTENANCE FOR CONTINUOUSLY OPERATING REPAIRABLE EQUIPMENT

by

S.H. Sim and J. Endrenyi

A minimal maintenance model is developed for repairable continuously operating devices whose conditions deteriorate with the time in service. The devices fail at the end of the deterioration process and can also fail randomly at any time. Optimal maintenance intervals are obtained by minimizing the total unavailability of the device due to maintenance, random failures and deterioration failures. The dependence of optimal maintenance rate on several parameters has been explored. The results may be useful for the assessment of maintenance policies for several power system components such as circuit breakers, transformers, and coal pulverizers.

INTRODUCTION

Maintenance policies, their selection and effects, have been studied in great detail for non-repaired components and systems. In such cases, preventive maintenance takes the form of a planned replacement as opposed to unplanned replacement which occurs after a failure. An optimal maintenance policy will minimize the total cost of planned and unplanned replacement.

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Much less is known about preventive maintenance in the presence of repair. Here an optimal policy minimizes the combined effect of repair and maintenance. Obviously, infrequent maintenance results in more frequent failures and an increased number of repair outages while frequent maintenance increases the number of outages on its own account; clearly, the two effects must be balanced. The result, determined by the optimal reliability (lowest unavailability) of the device, is the optimal frequency at which maintenance ought to be performed.

At a meeting with Transmission Stations Department [1], the need for a mathematical approach that can be used in shaping maintenance policies was acknowledged and Research was encouraged to explore the subject. Some of the conclusions reached at the meeting are the following.

- Computations of optimum maintenance intervals from realistic input data can be useful.
- In some instances, maintenance intervals were lengthened in the past with no apparent adverse effects. But no 'hard numbers' were collected and analyzed.
- Deterioration is a very real factor. It may be brought about by ageing or by the number of operations (eg. circuit breaker).
- Random failures do occur. Some of them are caused by external elements. It may be possible to screen them out from outage reports.
- In general, random failures are easier to repair than those caused by ageing.
- Priority should be given to studies of components which carry large portions of the maintenance budget so that potential savings would be the highest.

To follow up on these points, several mathematical models for maintenance were developed, solved, and the results analyzed. This report contains a brief description of the models and the results. A more detailed description is given in two papers submitted to the IEEE, the first of which is attached as an Appendix. Because of its mainly mathematical content, the second paper [2] is not, but its findings are discussed in this report. Interested readers may write to the authors for copies.

MODEL 1

This model, shown in Figure 1, is described in more detail in the Appendix. For definitions, assumptions, and notation, the reader is also referred to the Appendix. Here only a short description follows.

The state D_1 is the normal (new) state of the device. The process of deterioration is in k stages, and is represented by the states D_2, $D_3, \ldots, D_k$ where the device is still working; the last step of deterioration results in failure (F_1). Maintenance can be initiated out of any one

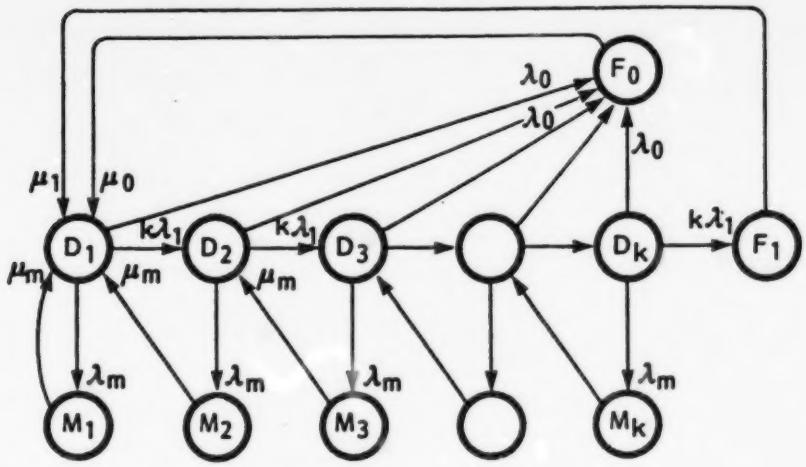

Figure 1

Maintenance model with both deterioration and random failures

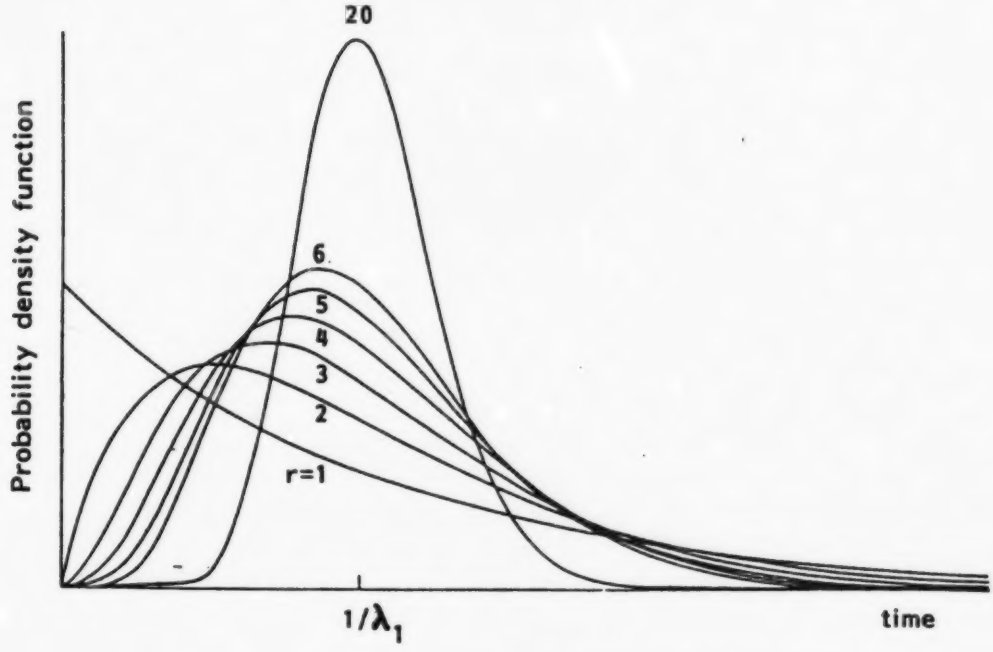

Figure 2

The family of Erlang distributions for the same mean time to deterioration failure ($1/\lambda_1$)

of the states $D_1,\ldots,D_k$, and maintenance started in D_i and performed in state M_i will result in an improvement which puts the device into state D_{i-1}, with the exception of maintenance started in D_1 which has no effect. This type of maintenance activity is called minimal maintenance (see Appendix). The failure state F_0 represents the random failures which can occur at any time and, therefore, state F_0 can be reached from any of the $D_1,\ldots, D_k$ states at the uniform rate of λ_0.

The model can be solved through Gauss-Seidel iterations, or by using the algorithm described in [2].

MODEL 2

One shortcoming of Model 1 is that it assumes an exponential distribution for the times to maintenance - an assumption which is seldom realistic. In Model 2, this constraint is relaxed [2]. A choice of distributions for the times to maintenance is achieved by breaking up the one-step transition to the states M_i of Model 1 into r stages, just as the deterioration process is broken up into k stages. It can be shown that the total times to maintenance will have an Erlang distribution, instead of the exponential in the r=1 case. It is easy to arrange that the mean of all of these distributions should always be the same $(1/\lambda_1)$.

The family of Erlang distributions is illustrated in Figure 2. It can be observed that as r increases, the distribution becomes narrower; in fact, if $r \to \infty$, the variability is entirely removed and the times to maintenance become deterministic (always the same). Note that the intermediate stages leading to the maintenance state have no physical meaning and serve only as a mathematical device to allow the use of simple Markovian solution techniques.

CONCLUSIONS

1. The concepts used in the proposed models correlate quite well with those used in the Hydro manual 'Electrical Maintenance Procedures' [3]. For example, 'minimal maintenance' in the models can be translated into procedures P1 to P3 in the manual, 'maintenance' into procedures P4 to P6, and so on.

2. The results of this study can be used for the reassessment of present maintenance policies for several power system components such as circuit breakers and transformers.

3. The analysis in the Appendix shows that for preventive maintenance (whether or not minimal) to be effective, the deterioration-originated failures must be the dominant component in the failure mix. Random failures cannot be prevented through maintenance.

4. One of the best methods to increase the optimal time to maintenance (reduce the need for maintenance) is to reduce the duration of repairs after failures due to deterioration.

5. The sensitivity of the component unavailability (combined repair and maintenance costs) to performing maintenance less frequently than optimal is generally low. More frequent maintenance may, however, significantly increase unavailability and costs.

6. The Erlang distribution introduced in Reference [2] provides a realistic representation for the times to maintenance. This distribution approaches a single (deterministic) value as the parameter r increases. If the intention is to consider deterministic times to maintenance, however, it may not be necessary to choose a very high value for r; in fact, it can be as low as 3 in most cases [2]. If deterioration failures dominate, the use of Model 1 (r=1) may be appropriate.

RECOMMENDATIONS

1. It is recommended that the models be tested on selected power system components. To do so, the cooperation of interested departments, such as the Transmission Stations Department, is needed in the following areas:

   - to choose appropriate components for the studies;

   - to provide data for the models, including values for the mean time to random failures ($1/\lambda_0$), the mean time to deterioration failures if no maintenance is performed ($1/\lambda_1$), the mean repair time after random failures ($1/\mu_0$), the same after deterioration failures ($1/\mu_1$), and the mean duration of minimal maintenance ($1/\mu_m$).

2. The models should be further refined to include the effects of major maintenance/overhauls.

Approved:

J.G. Cassan
Manager
Operations Research Dept

Submitted:

S.H. Sim
Engineer
Reliability & Statistics Section

J. Endrenyi
Supervising Engineer
Reliability & Statistics Section

SHS/JE:cmh

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APPENDIX

RELIABILITY OPTIMIZATION FOR CONTINUOUSLY OPERATING EQUIPMENT WITH MAINTENANCE AND REPAIR

J. Endrenyi S.H. Sim

Ontario Hydro
Toronto, Canada

Abstract - The study of preventive maintenance policies for components with repair after failures is a relatively unexplored area. In this paper, the failure/repair/maintenance sequences are described through a Markov process. The model used accounts for both random failures and failures due to deterioration, neither negligible in practical cases. Optimal maintenance intervals are derived by determining the point where the total unavailability of a component, due to repairs and maintenance, is at a minimum. The dependence of optimal maintenance rate on several parameters is explored. Results may assist in the reassessment of maintenance policies for power system components such as circuit breakers and transformers.

1 INTRODUCTION

Maintenance policies, their selection and effects, have been studied in great detail for non-repaired components and systems [1,2,3]. In such cases, preventive maintenance takes the form of planned replacement, as opposed to unplanned replacement which occurs after a failure. An optimal maintenance policy will minimize the total cost of planned and unplanned replacement.

Much less is known about preventive maintenance in the presence of repair. Here an optimal policy minimizes the combined effect of repair and maintenance. Obviously, infrequent maintenance results in more frequent failures and an increased number of repair outages while frequent maintenance increases the number of outages on its own account; clearly, the two effects must be balanced. The result, determined by the optimal reliability (lowest unavailability) of the device, is the optimal frequency at which maintenance ought to be performed. The basic assumption is that maintenance can be performed cheaper and faster than repair.

In this paper, a Markov model is proposed for describing the maintenance/repair process; optimization is carried out on the steady-state solution of the model. In developing the model, it is assumed that both random failures and failures due to deterioration can occur. Experience with many power system components indicates that neither of these effects can be neglected. The results of this study can be used for the reassessment of present maintenance policies for several power system components such as circuit breakers and transformers.

2 DEFINITIONS

Deterioration - A process where the condition of a device gradually (and measurably) worsens. If left unattended, the process will lead to deterioration failure.

Random Failure - A failure which can occur any time and which, it is assumed, does occur at constant rate.

Repair - An activity where a device is restored to working condition after a failure rendered it inoperative. If not indicated otherwise, it is assumed that after repair the device is as good as new.

Maintenance - An activity undertaken regularly at preselected (but not necessarily identical) intervals while the device is satisfactorily operating, to reduce or completely eliminate the deterioration accumulated.

Minimal Maintenance - A maintenance activity of limited effort and effect. If deterioration is modeled as occurring in discrete steps, minimal maintenance is assumed to set back the process by one step.

3 ASSUMPTIONS

(a) As already mentioned, both random and deterioration failures are modeled.

(b) Deterioration is modeled as occurring in k discrete steps. The times spent in each stage of deterioration are exponentially distributed with an identical mean of $1/k\lambda_1$.

(c) The times to random failure are exponentially distributed with a mean of $1/\lambda_0$.

(d) Repair results in a condition of the device, which is as good as new.

(e) Repair durations are exponentially distributed, with a mean of $1/\mu_0$ after random failures and a mean of $1/\mu_1$ after deterioration failures.

(f) Maintenance throws the deterioration process back by one step (minimal maintenance).

(g) Times to maintenance* have an exponential distribution with a mean of $1/\lambda_m$.

(h) Maintenance durations are exponentially distributed with a mean of $1/\mu_m$.

(i) The mean maintenance duration is smaller than either of the mean repair durations.

* In the rest of the paper, and if it does not cause confusion, "minimal maintenance" will simply be termed "maintenance".

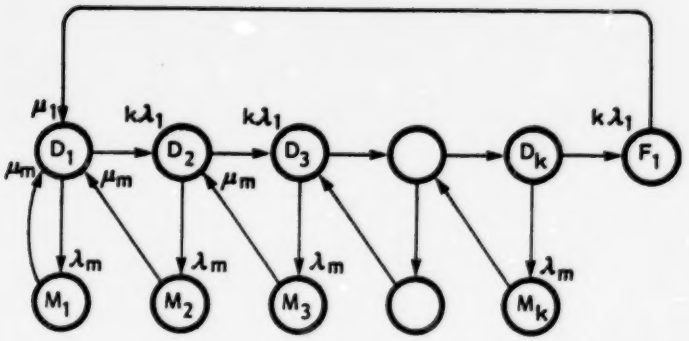

Figure 1

Maintenance model with no random failures

(j) Repair and maintenance have the same cost per unit time, and costs are proportional to the repair and maintenance durations. In that case, reliability optimization directly translates into cost optimization.

4 MODEL A - NO RANDOM FAILURES

In order to better understand the modeling of the deterioration and minimal maintenance processes and the method of optimization, a model omitting random failures is discussed first. The state-space diagram is shown in Figure 1. The state D_1 is the normal (new) state of the device. The process of deterioration is in k stages, and is represented by the states $D_2, D_3, \ldots, D_k$ where the device is still working; the last step of deterioration results in failure (F_1). By assumption (b), the rate of reaching the next stage of deterioration out of any given stage is $k\lambda_1$. Maintenance can be initiated out of any one of the states $D_1, \ldots, D_k$, and, according to assumption (f), maintenance started in D_i and performed in state M_i will result in an improvement which puts the device into state D_{i-1}, with the exception of maintenance started in D_1 which has no effect.

In solving the model, the equations for the long-term state probabilities can be routinely produced. Using mathematical induction then, it is easy to derive formulae for the state probabilities. These solutions are given in the Appendix.

The probability P_F that the device is not in service because of being either repaired (after failure) or maintained is

$$P_F = P_{f_1} + \sum_{i=1}^{k} Q_i$$

where P_{f_1} is the probability of the failure state F_1, Q_i are the probabilities of the corresponding M_i states, and k is the number of stages in the deterioration process.

By minimizing P_F with respect to $1/\lambda_m$ the optimal value of the mean time to minimal maintenance can be determined. An example is shown in Figure 2, where the assumed values of the various parameters are indicated in the diagram. Under the circumstances, the optimal value of the mean time to minimal maintenance is about 170 days, and the minimal value of P_F about 0.006. Note that the curve is quite flat on the right-hand side of the minimum and becomes steep on the left-hand side indicating that maintenance at a frequency lower than optimal would carry much less cost penalty than maintenance at a frequency higher than optimal.

It should be observed that only if failure can be modeled as the culmination of a multi-stage deterioration process can preventive maintenance be useful. In the usual two-state representation of a component (in service, in repair) with the in-service times exponentially distributed, the probability of impending failure (hazard rate) is the same at any time, and will not be influenced by maintenance. In contrast, it can be shown [4,5] that the times to failure when the in-service period is made up of stages (with exponentially distributed times between consecutive stages) have a gamma (more precisely: Erlang) distribution with hazard rates which in practical cases increase with time; now maintenance which restores the device to an earlier stage in the process will be useful because the probability of impending failure will thereby be reduced. Note that if, in a k-stage deterioration process, the rate of transition between the stages is uniformly $k\lambda_1$, the overall failure rate is λ_1.

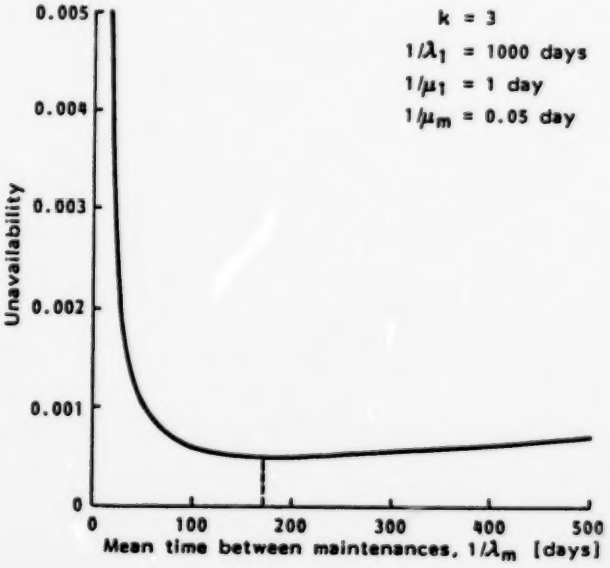

Figure 2

The effect of maintenance interval on the unavailability of a device

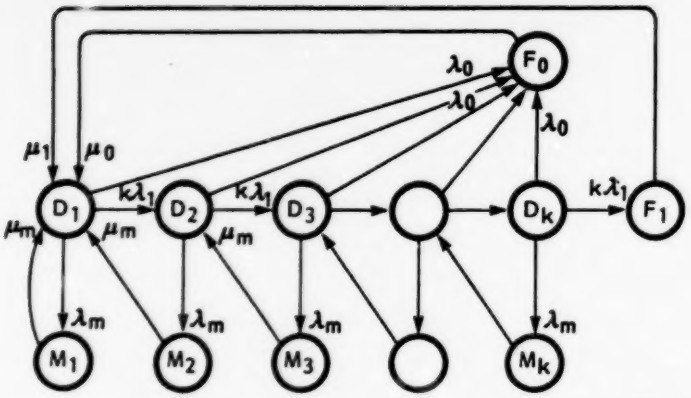

Figure 3

Maintenance model with both deterioration and random failures

5 MODEL B - RANDOM FAILURES INCLUDED

The state-space diagram for this model is illustrated in Figure 3. A comparison with the diagram in Figure 1 shows that now the failure state F_0 is added; this represents the random failures which can occur at any time and, therefore, state F_0 can be reached from any of the $D_1,\ldots,D_k$ states at the uniform rate of λ_0. Note that random failure cannot occur while maintenance is performed on the device. Repair after a random failure is completed at a rate of μ_0.

While it would be quite possible to use only a single failure state in the model, the distinction is made in order to accommodate two different mean repair times for the two failure modes. It is not unrealistic to assume, for example, that random failures take shorter time to repair than deterioration-originated failures.

Once again, the equations for the long-term state probabilities can be routinely derived from the state-space diagram. They are given in the Appendix. These equations can be easily solved by using Gauss-Seidel iterations.

As before, the optimal maintenance rate can be obtained by minimizing the total outage probability P_F. In the following, the dependence of this optimal λ_m value on the various input parameters will be investigated.

6 DISCUSSION

It is of interest to find out how the optimal time to maintenance (OTTM) depends on such parameters as the mean time to deterioration-originated failures (TTDF), the mean time to random failures (TTRF), the mean repair time for deterioration-originated failures (DT), for random failures (RT), and the mean maintenance duration (MT). There are simple connections between these quantities and the rates in Figures 1 and 3; as a reminder, they are listed in the following:

$$TTDF = 1/\lambda_1$$
$$TTRF = 1/\lambda_0$$
$$DT = 1/\mu_1$$
$$RT = 1/\mu_0$$
$$MT = 1/\mu_m$$

and OTTM is the optimal value of $1/\lambda_m$.

The relation between the OTTM and some of these quantities is depicted in the diagram of Figure 4. Here OTTM, given in multiples of TTDF, is plotted as a function of the ratio DT/MT. Low OTTM values indicate a higher (more frequent) need for maintenance, whereas high OTTM values mean an infrequent need for it. It is well to remember, particularly in the latter case, that increasing the maintenance frequency beyond what is optimally required would be ineffective and, in fact, counter-productive.

The first thing that is apparent from the diagram in Figure 4 is that OTTM decreases as DT increases. This trend can be easily understood if one considers that low DT values mean failures with fast repairs where prevention by maintenance is less important than in the case of failures with more time-consuming repairs. At DT/MT = 1 maintenance would take as long as repair after failure, obviating the need for maintenance.

By plotting these curves for several MT values, it can be shown that only the DT/MT ratio is what matters; one obtains the same OTTM/TTDF values for, say, DT = 10 days and MT = 1 day as for, say, DT = 5 days and MT = 0.5 day.

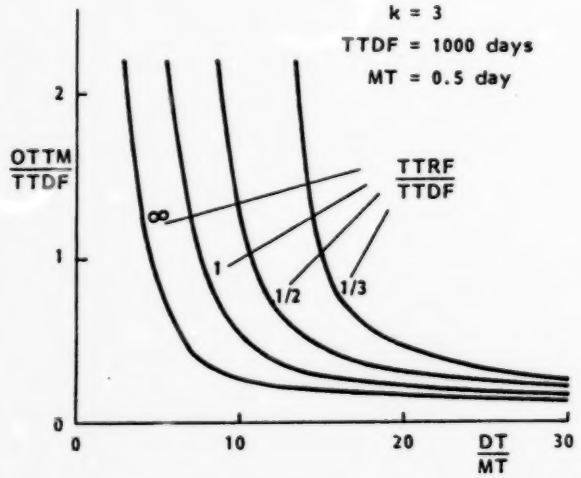

Figure 4

Dependence of the optimal time to maintenance (OTTM) on the various failure parameters

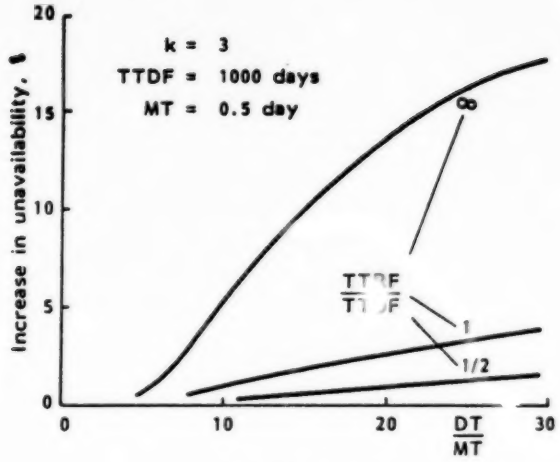

Figure 5

Increase in unavailability if maintenance is performed at intervals twice the optimal, in terms of the mean repair time after failures due to deterioration

If TTRF (and, therefore, TTRF/TTDF) approaches infinity, random failures become negligible. This is the "worst case" where the most frequent maintenance is needed. Conversely, if random failures dominate, the need for maintenance reduces; if only random failures exist, maintenance has no effect at all (as already mentioned) and, therefore, there is no need for it. For the same reason, the parameter RT has only a limited influence on the curves in Figure 4.

Note that if DT is large, random failures will reduce the need for maintenance at a slower rate as TTRF/TTDF decreases. The effect of failures due to deterioration depends not only on their frequency but also on their duration.

In connection with Figure 2 it was observed that the effect of choosing a maintenance frequency somewhat lower than optimal is relatively minor. To explore this quantitatively, the changes in component unavailability (outage probability) were calculated if the mean time to maintenance (TTM) is increased from the optimal to twice the optimal value. The results are shown in Figure 5, in terms of the same parameters that were used in Figure 4. In the worst case, when there are no random failures, changes are the highest but still within 20% in the range shown; they reduce rapidly as DT reduces. If, as usual, the failures are a mix of deterioration and random types, the effect of reducing the maintenance frequency to about half its optimal value is markedly diminished.

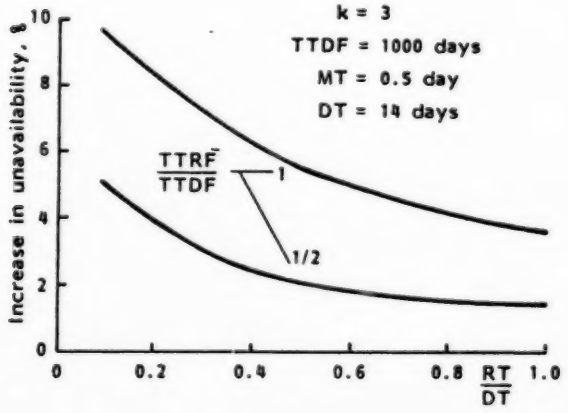

Figure 6

Increase in unavailability if maintenance is performed at intervals twice the optimal, in terms of the mean repair time after random failures

Figure 6 illustrates the changes in TTM if the mean duration of random failures, RT, is varied. As can be seen, the effect reduces as RT increases, which is in keeping with what one would expect since longer random failure durations increase the influence of the random failures.

7 CONCLUSIONS AND FUTURE RESEARCH

From the foregoing, the following main conclusions can be drawn.

1. A multistate model is proposed to describe the effect of preventive maintenance on the operation of repairable components. The inclusion of failures occurring both randomly and after a process of deterioration adds to the realism of the model. The solution of the model for finding the optimal mean time to maintenance (OTTM) is fairly simple.

2. For preventive maintenance (whether or not minimal) to be effective, the deterioration-originated failures must be the dominant component in the failure mix. Random failures cannot be prevented through maintenance.

3. One of the best methods to increase the OTTM (reduce the need for maintenance) is to reduce the duration of repairs after failures due to deterioration.

4. The sensitivity of the component unavailability (combined repair and maintenance costs) to performing maintenance less frequently than optimal is generally low. More frequent maintenance may, however, significantly increase unavailability and costs.

Further research is needed in the following areas. Some of it is already in progress.

- Assumption (g) which requires that the times to maintenance have an exponential distribution ought to be relaxed. Maintenance is a planned activity and the exponential is a poor distribution to describe the inter-maintenance times. Presently models are under study which do not require this assumption.

- It has been suggested that the effect of major overhauls involving the re-building of a component be also included in the model.

- Methods for identifying from data banks the failure and repair rates used in the models must be developed.

- Application studies must be carried out. Quite obviously, such studies must start with the collection of the failure, repair and maintenance data needed in the given application.

APPENDIX

Equations for the Long-Term State Probabilities

Based on the state-space diagram in Figure 3, it is quite straightforward to derive the equations for the long-term state probabilities. The following notation is used:

P_i are the probabilities of states D_i

Q_i are the probabilities of states M_i

P_{F_1} is the probability of state F_1

P_{f_0} is the probability of state F_0

The state transition rates are explained in conjunction with Figures 1 and 3. The state probability equations (with random failures included) then become:

$$(\lambda_0+k\lambda_1+\lambda_m)P_1 = \mu_0 P_{f_0} + \mu_1 P_{f_1} + \mu_2(Q_1+Q_2)$$

$$(\lambda_0+k\lambda_1+\lambda_m)P_i = k\lambda_1 P_{i-1} + \mu_2 Q_{i+1} \qquad 2 \le i \le k-1$$

$$(\lambda_0+k\lambda_1+\lambda_m)P_k = k\lambda_1 P_{k-1}$$

$$\mu_m Q_i = \lambda_m P_i \qquad 1 \le i \le k$$

$$\mu_1 P_{f_1} = k\lambda_1 P_k$$

$$\mu_0 P_{f_0} = \lambda_0 \sum_{i=1}^{k} P_i$$

$$P_{f_1} + P_{f_0} + \sum_{i=1}^{k} P_i + \sum_{i=1}^{k} Q_i = 1$$

This set of equations can be solved by an algorithmic approach such as Gauss-Seidel iteration.

The equations for the model in Figure 1 (no random failures) can be easily derived from the above set by setting $\lambda_0 = \mu_0 = 0$. In this case, a simple solution can be obtained by mathematical induction:

$$P_{k-j} = \frac{1-\beta^{j+1}}{1-\beta} P_k \qquad 0 \le j \le k-1$$

$$Q_{k-j} = \rho_m P_{k-j}$$

$$P_{f_1} = k\rho_1 P_k$$

$$P_k = \left[\frac{1+\rho_m}{1-\beta}\left(k - \beta\frac{1-\beta^k}{1-\beta}\right) + k\rho_1\right]^{-1}$$

where $\rho_1 = \lambda_1/\mu_1$, $\rho_m = \lambda_m/\mu_m$ and $\beta = \lambda_m/k\lambda_1$.

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